ДЗ 3

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Содержание

1 2

1.1 a

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma \vdash f : \alpha \to \beta \to \gamma} \quad \overline{\Gamma \vdash a : \alpha}}{\Gamma \vdash f\ a : \beta \to \gamma} \quad \overline{\Gamma \vdash b : \beta}}{\Gamma \vdash f\ a\ b : \gamma}}{f : (\alpha \to \beta \to \gamma), a : \alpha \vdash \lambda a.f\ a\ b : \beta \to \gamma}}{f : (\alpha \to \beta \to \gamma) \vdash \lambda b.\lambda a.f\ a\ b : \alpha \to (\beta \to \gamma)}}{\vdash \lambda f.\lambda b.\lambda a.f\ a\ b : (\alpha \to \beta \to \gamma) \to (\alpha \to \beta \to \gamma)}$$

где $\Gamma = f : \alpha \to \beta \to \gamma, a : \alpha, b : \beta$

```haskell
task :: (a -> b -> c) -> (b -> a -> c)
task f b a = f a b

main = do
  putStrLn "works!"
```

1.2 b

$$\dfrac{\dfrac{\dfrac{\dfrac{}{f:\alpha\to\alpha\to\beta, a:\alpha\vdash f:\alpha\to\alpha\to\beta}\quad\dfrac{}{f:\alpha\to\alpha\to\beta, a:\alpha\vdash a:\alpha}}{f:\alpha\to\alpha\to\beta, a:\alpha\vdash f\ a:\alpha\to\beta}\quad\dfrac{}{f:\alpha\to\alpha\to\beta, a:\alpha\vdash a:\alpha}}{f:\alpha\to\alpha\to\beta, a:\alpha\vdash f\ a\ a:\beta}}{f:\alpha\to\alpha\to\beta\vdash\lambda a.f\ a\ a:\alpha\to\beta}}{\vdash\lambda f.\lambda a.f\ a\ a:(\alpha\to\alpha\to\beta)\to(\alpha\to\beta)}$$

```haskell
task :: (a -> a -> b) -> (a -> b)
task f a = f a a

main = do
  putStrLn "works!"
```

2 5

2.1 a

2.1.1 i

$$\alpha\vee\beta\to\neg(\neg\alpha\&\neg\beta)$$

$$\alpha\vee\beta\to((\alpha\to\bot)\&(\beta\to\bot))\to\bot$$

$$\lambda c.\lambda p.\textbf{case}\ c\ (\lambda a.\pi_l\ p\ a)\ (\lambda b.\pi_r\ p\ b)$$

2.1.2 ii

```haskell
task :: Either a b -> ((a -> Void), (b -> Void)) -> Void
task (Left a) (fa, _) = fa a
task (Right b) (_, fb) = fb b

main = do
  putStrLn "works!"
```

2.1.3 iii

$$\dfrac{\dfrac{}{\Gamma\vdash c:\alpha\vee\beta}\quad\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, a:\alpha\vdash p:(\alpha\to\bot)\&(\beta\to\bot)}}{\Gamma, a:\alpha\vdash\pi_l\ p:\alpha\to\bot}\quad\dfrac{}{\Gamma, a:\alpha\vdash a:\alpha}}{\Gamma, a:\alpha\vdash\pi_l\ p\ a:\bot}}{\Gamma\vdash\lambda a.\pi_l\ p\ a:\alpha\to\bot}\quad\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, b:\beta\vdash p:(\alpha\to\bot)\&(\beta\to\bot)}}{\Gamma, b:\beta\vdash\pi_r\ p:\beta\to\bot}\quad\dfrac{}{\Gamma, b:\beta\vdash b:\beta}}{\Gamma, b:\beta\vdash\pi_r\ p\ b:\bot}}{\Gamma\vdash\lambda b.\pi_r\ p\ b:\beta\to\bot}}{\dfrac{\dfrac{\dfrac{\Gamma\vdash\textbf{case}\ c\ (\lambda a.\pi_l\ p\ a)\ (\lambda b.\pi_r\ p\ b):\bot}{c:(\alpha\vee\beta)\vdash\lambda p.\textbf{case}\ c\ (\lambda a.\pi_l\ p\ a)\ (\lambda b.\pi_r\ p\ a):((\alpha\to\bot)\&(\beta\to\bot))\to\bot}}{\vdash\lambda c.\lambda p.\textbf{case}\ c\ (\lambda a.\pi_l\ p\ a)\ (\lambda b.\pi_r\ p\ b):(\alpha\vee\beta)\to((\alpha\to\bot)\&(\beta\to\bot))\to\bot}}$$

, где $\Gamma = c:(\alpha\vee\beta), p:((\alpha\to\bot)\&(\beta\to\bot))$

2.2 b

2.2.1 i

$$\alpha\&\beta\to\neg(\neg\alpha\vee\neg\beta)$$

$$\alpha\&\beta\to((\alpha\to\bot)\vee(\beta\to\bot))\to\bot$$

$$\lambda p.\lambda c.\textbf{case}\ c\ (\lambda f.f\ (\pi_l\ p))\ (\lambda f.f\ (\pi_r\ p))$$

2.2.2 ii

```haskell
task :: (a, b) -> Either (a -> Void) (b -> Void) -> Void
task (a, _) (Left fa) = fa a
task (_, b) (Right fb) = fb b

main = do
  putStrLn "works!"
```

2.2.3 iii

$$\dfrac{\dfrac{}{\Gamma \vdash c : (\alpha \to \bot) \vee (\beta \to \bot)} \quad \dfrac{\dfrac{}{\Gamma, f : \alpha \to \bot \vdash f : (\alpha \to \bot)} \quad \dfrac{}{\Gamma, f : \alpha \to \bot \vdash \pi_l\ p : \alpha}}{\dfrac{\Gamma, f : \alpha \to \bot \vdash f\ (\pi_l p) : \bot}{\Gamma \vdash \lambda f.f\ (\pi_l\ p) : (\alpha \to \bot) \to \bot}} \quad \dfrac{\dots}{\Gamma \vdash \lambda f.f\ (\pi_r\ p) : (\beta \to \bot) \to \bot}}{\dfrac{\Gamma \vdash \mathbf{case}\ c\ (\lambda f.f\ (\pi_l\ p))\ (\lambda f.f\ (\pi_r\ p))}{\dfrac{p : \alpha \& \beta \vdash \lambda c.\mathbf{case}\ c\ (\lambda f.f\ (\pi_l\ p))\ (\lambda f.f\ (\pi_r\ p)) : ((\alpha \to \bot) \vee (\beta \to \bot)) \to \bot}{\vdash \lambda p.\lambda c.\mathbf{case}\ c\ (\lambda f.f\ (\pi_l\ p))\ (\lambda f.f\ (\pi_r\ p)) : \alpha \& \beta \to ((\alpha \to \bot) \vee (\beta \to \bot)) \to \bot}}$$

$$\dfrac{\dfrac{}{\Gamma, f : \beta \to \bot \vdash f : \beta \to \bot} \quad \dfrac{}{\Gamma, f : \beta \to \bot \vdash \pi_r\ p : \beta}}{\dfrac{\Gamma, f : \beta \to \bot \vdash f\ (\pi_r\ p) : \bot}{\Gamma \vdash \lambda f.f\ (\pi_r\ p) : (\beta \to \bot) \to \bot}}$$

, где $\Gamma = p : \alpha \& \beta, c : (\alpha \to \bot) \vee (\beta \to \bot)$

2.3 c

2.3.1

$$a \& (\beta \vee \gamma) \to (\alpha \& \beta) \vee (\alpha \& \gamma)$$

$$\lambda p.\mathbf{case}\ (\pi_r\ p)\ (\lambda b.\mathbf{in}_l\ \langle \pi_l\ p, b\rangle)\ (\lambda c.\mathbf{in}_r\ \langle \pi_l\ p, c\rangle)$$

2.3.2

```haskell
task :: (a, Either b c) -> Either (a, b) (a, c)
task (a, Left b) = Left (a, b)
task (a, Right c) = Right (a, c)
```

2.3.3

$$\dfrac{\dfrac{}{p : \alpha \& (\beta \vee \gamma) \vdash p : \alpha \& (\beta \vee \gamma)} \quad \dfrac{\dfrac{\dfrac{\dfrac{}{p : \alpha \& (\beta \vee \gamma), b : \beta \vdash p : \alpha \& (\beta \vee \gamma)}}{p : \alpha \& (\beta \vee \gamma), b : \beta \vdash \pi_l\ p : \alpha} \quad \dfrac{}{p : \alpha \& (\beta \vee \gamma), b : \beta \vdash b : \beta}}{p : \alpha \& (\beta \vee \gamma), b : \beta \vdash \langle \pi_l\ p, b\rangle : \alpha \& \beta}}{p : \alpha \& (\beta \vee \gamma), b : \beta \vdash \mathbf{in}_l\ \langle \pi_l\ p, b\rangle : (\alpha \& \beta) \vee (\alpha \& \gamma)}}{p : \alpha \& (\beta \vee \gamma) \vdash \lambda b.\mathbf{in}_l\ \langle \pi_l\ p, b\rangle : \beta \to (\alpha \& \beta) \vee (\alpha \& \gamma)}\ (1)$$

$$\dfrac{p : a \& (\beta \vee \gamma) \vdash \mathbf{case}\ (\pi_r\ p)\ (\lambda b.\mathbf{in}_l\ \langle \pi_l\ p, b\rangle)\ (\lambda c.\mathbf{in}_r\ \langle \pi_l\ p, c\rangle) : (a \& \beta) \vee (a \& \gamma)}{\vdash \lambda p.\mathbf{case}\ (\pi_r\ p)\ (\lambda b.\mathbf{in}_l\ \langle \pi_l\ p, b\rangle)\ (\lambda c.\mathbf{in}_r\ \langle \pi_l\ p, c\rangle) : a \& (\beta \vee \gamma) \to (a \& \beta) \vee (a \& \gamma)}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{p : \alpha \& (\beta \vee \gamma), c : \gamma \vdash p : \alpha \& (\beta \vee \gamma)}}{p : \alpha \& (\beta \vee \gamma), c : \gamma \vdash \pi_l\ p : \alpha} \quad \dfrac{}{p : \alpha \& (\beta \vee \gamma), c : \gamma \vdash c : \gamma}}{p : \alpha \& (\beta \vee \gamma), c : \gamma \vdash \langle \pi_l\ p, c\rangle : (\alpha \& \gamma)}}{p : \alpha \& (\beta \vee \gamma), c : \gamma \vdash \mathbf{in}_r\ \langle \pi_l\ p, c\rangle : (\alpha \& \beta) \vee (\alpha \& \gamma)}}{p : \alpha \& (\beta \vee \gamma) \vdash \lambda c.\mathbf{in}_r\ \langle \pi_l\ p, c\rangle : \gamma \to (\alpha \& \beta) \vee (\alpha \& \gamma)}\ (1)$$

2.4 d

2.4.1 i

$$(\alpha \to \beta) \to (\neg\beta \to \neg\alpha)$$

$$(\alpha \to \beta) \to (\beta \to \bot) \to \alpha \to \bot$$

$$\lambda f.\lambda b.\lambda a.b\ (f\ a)$$

2.4.2 ii

```
task :: (a -> b) -> (b -> Void) -> a -> Void
task f fb a = fb (f a)

main = do
  putStrLn "works!"
```

2.4.3 iii

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash b : \beta \to \bot} \quad \dfrac{\dfrac{}{\Gamma \vdash f : \alpha \to \beta} \quad \dfrac{}{\Gamma \vdash a : \alpha}}{\Gamma \vdash f\ a : \beta}}{\Gamma \vdash b\ (f\ a) : \bot}}{f : \alpha \to \beta, b : \beta \to \bot \vdash \lambda a.b\ (f\ a) : \alpha \to \bot}}{f : \alpha \to \beta \vdash \lambda b.\lambda a.b\ (f\ a) : (\beta \to \bot) \to \alpha \to \bot}}{\vdash \lambda f.\lambda b.\lambda a.b\ (f\ a) : (\alpha \to \beta) \to (\beta \to \bot) \to \alpha \to \bot}$$

, где $\Gamma = f : \alpha \to \beta, b : \beta \to \bot, a : \alpha$

2.5 e

2.5.1 i

$$\alpha \to \neg\alpha \to \beta$$

$$\alpha \to (\alpha \to \bot) \to \beta$$

$$\lambda a.\lambda f.f\ a$$

2.5.2 ii

```
task :: a -> (a -> Void) -> b
task a f = f a

main = do
  putStrLn "works!"
```

2.5.3 iii

$$\dfrac{\dfrac{\dfrac{\dfrac{}{a : \alpha, f : \alpha \to \bot \vdash f : \alpha \to \bot} \quad \dfrac{}{a : \alpha, f : \alpha \to \bot \vdash a : \alpha}}{a : \alpha, f : \alpha \to \bot \vdash f\ a : \bot}}{a : \alpha, f : \alpha \to \bot \vdash f\ a : \beta}}{a : \alpha \vdash \lambda f.f\ a : (\alpha \to \bot) \to \beta}}{\vdash \lambda a.\lambda f.f\ a : \alpha \to (\alpha \to \bot) \to \beta}$$