ДЗ 1

Ilya Yaroshevskiy

14 сентября

Содержание

1 1

1.1 a

$$\lambda z.\lambda x.abc\lambda y.def$$

$$\Downarrow$$

$$\lambda z.(\lambda x.(((ab)c)(\lambda y.((de)f))))$$

1.2 b

$$(\lambda f.((\lambda x.(f(f(x(\lambda z.(zs))))))z))$$

$$\Downarrow$$

$$\lambda f.(\lambda x.f(f(x\lambda z.zs)))z$$

2 2

2.1 a

$$\begin{aligned} T\ F \to_\beta \\ (\lambda a.\lambda b.a)(\lambda a.\lambda b.b) \to_\beta \\ (\lambda b.\lambda a.\lambda c.c) \end{aligned}$$

2.2 b

$$\begin{aligned} (T\ \ Not\ (\lambda t.t))\ F \to_\beta \\ ((\lambda a.\lambda b.a)\ (\lambda x.x\ F\ T)(\lambda t.t))\ (\lambda a.\lambda b.b) \to_\beta \\ ((\lambda b.\lambda x.x\ F\ T)\ (\lambda t.t))\ (\lambda a.\lambda b.b) \to_\beta \\ (\lambda x.x\ F\ T)(\lambda a.\lambda b.b) \to_\beta \\ (\lambda a.\lambda b.b)\ F\ T \to_\beta \\ T \to_\beta \\ \lambda a.\lambda b.a \end{aligned}$$

2.3 c

$$\begin{aligned} \&\quad (\ \&\quad F\ F)\ T \to_\beta \\ (\lambda x.\lambda y.x\ y\ F)\ (\ \&\quad F\ F)\ T \to_\beta \\ (\ \&\quad F\ F)\ T\ F \to_\beta \\ ((\lambda x.\lambda y.x\ y\ F)\ F\ F)\ T\ F \to_\beta \\ (F\ F\ F)\ T\ F \to_\beta \\ ((\lambda a.\lambda b.b)\ F\ F)\ T\ F \to_\beta \\ F\ T\ F \to_\beta \\ (\lambda a.\lambda b.b)\ T\ F \to_\beta \\ F \to_\beta \\ \lambda a.\lambda b.b \end{aligned}$$

3 3

3.1 a

$$Or = \lambda x.\lambda y.x\ T\ y$$

3.2 b

$$(\cdot) = \lambda x.\lambda y.\, Not(\ \&\quad x\ y)$$

3.3 c

$$Xor = \lambda x.\lambda y.x\ (y\ F\ T)\ (y\ T\ F)$$

4 4

4.1 a

$$Mul2 = \lambda n.(\cdot)\ \overline{2}$$

4.2 b

$$Pow = \lambda a.\lambda b.b\ ((\cdot)\ a)\ \overline{1}$$

$$Pow = \lambda a.\lambda b.ba$$

4.3 c

$$Odd = \lambda a.a\ (Xor\ T)\ F$$
$$Even = \lambda a.a\ (Xor\ T)\ T$$

4.4 d

$$IsZero = \lambda a.a(And\ F)T$$

5 5

5.1 a

$$\begin{aligned} PrL\ (MkPair\ a\ b) &\to_\beta \\ (\lambda p.p\ T)\ ((\lambda a.\lambda b.(\lambda x.x\ a\ b))\ a\ b) &\to_\beta \\ (\lambda p.p\ T)\ ((\lambda b.(\lambda x.x\ a\ b))\ b) &\to_\beta \\ (\lambda p.p\ T)\ (\lambda x.x\ a\ b) &\to_\beta \\ (\lambda x.x\ a\ b)\ (\lambda a.\lambda b.a) &\to_\beta \\ (\lambda a.\lambda b.a)\ a\ b &\to_\beta \\ (\lambda b.a)\ b &\to_\beta \\ a & \end{aligned}$$

5.2 b

$$(-1) = \lambda n.\lambda f.\lambda x.\ PrL\ (n\ (\lambda p.(PrR\ p)\ (MkPair\ (f\ (PrL\ p))\ T)\ (MkPair\ (PrL\ p)\ T))\ (MkPair\ x\ F))$$

5.3 c

$$(-) = \lambda a.\lambda b.b\ (-1)\ a$$

6 8

6.1 a

$$\begin{aligned} Y\ f &\to_\beta \\ \lambda f.(\lambda x.f\ (x\ x))\ (\lambda x.f\ (x\ x))\ f &\to_\beta \\ (\lambda x.f\ (x\ x))\ (\lambda x.f\ (x\ x)) &\to_\beta \\ f\ ((\lambda x.f\ (x\ x))\ (\lambda x.f\ (x\ x))) &=_\beta \\ f\ (Yf) & \end{aligned}$$