

# Практика 13

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## Содержание

**Задача 1.** Дана двумерная дискретная случайная величина

$\eta \backslash \xi$	1	2	3	5	$g_j$
0	0.1	0.1	0.1	0.1	0.4
2	0.15	0.15	0	0.5	0.35
4	0.05	0.05	0.1	0.05	0.25
$p_i$	0.3	0.3	0.2	0.2	$\sum = 1$

Решение.

$$\begin{array}{c|cccc} \xi & 1 & 2 & 3 & 5 \\ \hline p_i & 0.3 & 0.3 & 0.2 & 0.2 \end{array}$$

$$\begin{array}{c|ccc} \eta & 0 & 2 & 4 \\ \hline g_j & 0.4 & 0.35 & 0.25 \end{array}$$

$\xi|\eta = 0$ :

$$\begin{array}{c|cccc} \xi|\eta = 0 & 1 & 2 & 3 & 5 \\ \hline p_i & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{array}$$

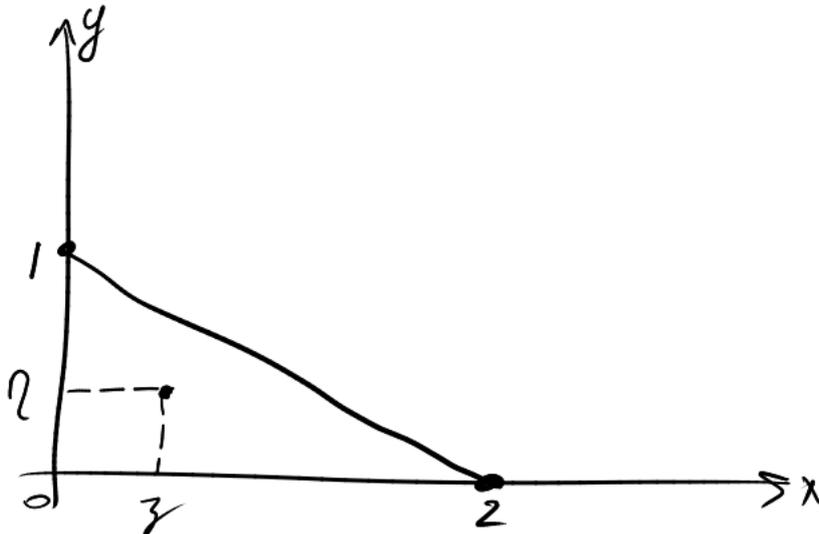
$$p(\xi = 1|\eta = 0) = \frac{0.1}{0.4} = \frac{1}{4}$$

$$E(\xi|\eta = 0) = 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{4} + 5 \cdot \frac{1}{4} = 2.75$$

$$\begin{array}{c|ccc} \eta|\xi = 1 & 0 & 2 & 4 \\ \hline g_i & \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \end{array}$$

$$E(\eta|\xi = 1) = 1 + \frac{2}{3} = \frac{5}{3}$$

**Задача 2.**



Решение.  $f_{\xi,\eta} = C$

$$\iint_D f(x, y) dx dy = \iint_D C dx dy = C \iint_D dx dy = C \cdot \int_D = 1 \implies f_{\xi,\eta}(x, y) = 1$$

Решение.

$$f_{\xi}(x) = \int_{-\infty}^{\infty} f_{\xi,\eta}(x, y) dy = \int_0^{1-\frac{x}{2}} dy = 1 - \frac{x}{2} \quad 0 \leq x \leq 2$$

$$f_{\eta}(y) = \int_{-\infty}^{\infty} f_{\xi,\eta}(x, y) dx = \int_0^{2-2y} dx = 2 - 2y \quad 0 \leq y \leq 1$$

$$E\xi = \int_{-\infty}^{\infty} x f_{\xi}(x) dx = \int_0^2 x(1 - \frac{x}{2}) dx = \frac{2}{3}$$

$$E\eta = \int_0^1 y(2 - 2y) dy = \frac{1}{3}$$

Решение.

$$f(y|x) = \frac{f_{\xi,\eta}(x, y)}{f_{\xi}(x)} = \frac{1}{1 - \frac{x}{2}} \quad 0 \leq x \leq 2$$

Решение.

$$\begin{aligned} E(\eta|\xi) &= \int_{-\infty}^{\infty} y \cdot f(y|x) dy = \int_0^{1-\frac{x}{2}} y \cdot \frac{1}{1 - \frac{x}{2}} dy = \frac{1}{1 - \frac{x}{2}} \cdot \frac{y^2}{2} \Big|_0^{1-\frac{x}{2}} = \\ &= \frac{1}{2} \cdot \frac{(1 - \frac{x}{2})^2}{(1 - \frac{x}{2})} = \frac{2 - x}{4} \quad 0 \leq x \leq 2 \\ E(\eta|\xi = 1) &= \frac{2 - 1}{4} = \frac{1}{4} \end{aligned}$$

**Задача 3.**  $\eta = g(\xi_1, \xi_2)$

$$\begin{aligned} F_{\eta}(z) &= \int_{D_z} f_{\xi_1, \xi_2}(x, y) dx dy \\ D_z &= \{(x, y) | g(x, y) < z\} \end{aligned}$$

Решение.

$$\begin{aligned} f_{\xi}(x) &= \begin{cases} 0 & x < 0 \\ e^{-x} & x \geq 0 \end{cases} \\ f_{\eta}(y) &= \begin{cases} 0 & y < 0 \\ e^{-y} & y \geq 0 \end{cases} \end{aligned}$$

Т.к. неравномерна, то при  $x, y \geq 0$

$$f_{\xi,\eta}(x, y) = f_{\xi}(x) f_{\eta}(y) = e^{-x-y}$$

Решение.

$$D_z = \{(x, y) | \frac{x}{y} < z\} = \{(x, y) | x < zy\}$$

— Прямая проходящая через начало координат

$$F_{\eta}(z) = \int_0^{\infty} dy \int_0^{zy} e^{-x-y} dx$$