Практика 9

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Содержание

$A \subset \mathbb{R}^m$

$$\int_A f d\lambda_M$$

$$\begin{aligned} &x_1 = r\cos\varphi_1 \\ &x_2 = r\sin\varphi_1\cos\varphi_2 \\ &\vdots \\ &x_{m-1} = r\sin\varphi_1\dots\sin\varphi_{m-2}\cos\varphi_{m-1} \\ &x_m = \dots\sin\varphi_{m-1} \end{aligned} \qquad J = r^{m-1}(\sin\varphi_1)^{m-2}\dots(\sin\varphi_{m-2})^1$$

Задача 1.

$$\int_0^a dx_1\int_0^{x_1}dx_2\int_0^{x_{n-1}} f dx_n = \int_0^a dx_n\int_{x_n}^a dx_{n-1}\int_{x_{n-1}}^a\cdot\int_{x_2}^2 dx_1$$

Задача 2.

$$\int_0^t dt_1\int_0^{t_1}dt_2\cdots\int_0^{t_{n-1}} f(t_1)f(t_2)\dots f(t_n)dt_n = \frac{1}{n!}\left(\int_0^t f(s)ds\right)^n$$

$$\int_0^t f(x_1)\int_0^{x_1} f(x_2)dx_2dx_1 \overset{?}{=\!=} \frac{1}{2}\left(\int_0^t f(x)dx\right)^2 = \frac{1}{2}F(t)^2$$

$$\int_0^t f(x_1)\int_0^{x_1} f(x_2)dx_2dx_1 = \int_0^t f(x_1)F(x_1)dx_1 = \int_0^t f(x_1)\cdot\frac{1}{(n-1)!}F(t)^{n-1} = \frac{1}{n!}F^n\Big|_0^t$$

Задача 3 (4204а)**.**

$$\int_0^1\cdots\int_0^1 (x_1^2+\dots+x_n^2)\,dx_1\dots dx_n$$

$$\int_0^1 x_k^2\,dx_k\iiint_{[0,1]} 1dx_{n-1} = \frac{1}{3}\mapsto\frac{n}{3}$$

Задача 4 (4207)**.**

$$S\sqrt{x_1+\dots+x_n}\,dx_1\dots dx_n$$

$$x_i\ge 0\quad x_1+\dots+x_n\le 1$$

Решение.

$$\left[\begin{aligned} &t_1 = x_1 \\ &t_2 = x_1+x_2 \\ &\vdots \\ &t_n = x_1+x_2+\dots+x_n \end{aligned}\right.$$

$$0\le t_1\le t_2\le\dots\le t_n\le 1$$

$$J = 1$$

$$I = \int_0^1\sqrt{t_n}dt_n\int_0^{t_n}dt_{n-1}\int_0^{t_{n-1}}dt_{n-2}\cdots\int_0^{t_2}dt_1$$

$$V_{n-1} = \frac{1}{(n-1)!}R^{n-1}$$

Задача 5 (4210)**.**

$$\frac{x_1^2}{a_1^2} + \dots + \frac{x_{n-1}^2}{a_{n-1}^2} = \frac{x_n^2}{a_n^2} \quad x_n = a_x$$

Решение. Конус. Замена в полярные

$$x_n = a_n - x$$

$$\left[\begin{array}{l} x_1 = a_1 r\cos\varphi_1 \\ \vdots \\ x_{n-1} = a_{n-1} r\sin\varphi_1 \dots \sin\varphi_{n-1} \end{array}\right.$$

$$V = \int_A 1 d\lambda_n = \int_0^1 dx \underbrace{\int_0^x dr \int_0^\pi d\varphi_1 \cdots \int_0^{2\pi} d\varphi_{n-2}}_{\lambda_{n-1}B(0,x)} \cdot a_1 a_2 \dots a_n 2^{n-2} (\sin\varphi_2)^{n-3} \dots (\sin\varphi_n - 3)^1 =$$

$$\lambda_m B(0,R) = \frac{\pi^{\frac{m}{2}} R^m}{\Gamma\left(\frac{m}{2}+1\right)}$$

$$= \prod a_i \cdot \int_0^1 dx \cdot \frac{\pi^{\frac{n-1}{2}} R^{n-1}}{\Gamma\left(\frac{n-1}{2}+1\right)} = \prod a_i \cdot \frac{\pi^{\frac{n-1}{2}}}{\Gamma\left(\frac{n-1}{2}+1\right)} \int_0^1 x^{n-1}\, dx = \frac{\pi^{\frac{n-1}{2}}}{\Gamma\left(\frac{n-1}{2}+1\right) n} \cdot a_1 a_2 \dots a_n$$

Задача 6 (4114)**.**

$$\int_0^x dx_1 \int_0^{x_1} dx_2 \cdots \int_0^{x_{n-1}} f(x_n)\, dx_n = \int_0^x f(u) \frac{(x-u)^{n-1}}{(n-1)!}\, du$$