

Практика 9

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Содержание

$A \subset \mathbb{R}^m$

$$\int_A f d\lambda_M$$

$$x_1 = r \cos \varphi_1$$

$$x_2 = r \sin \varphi_1 \cos \varphi_2$$

\vdots

$$x_{m-1} = r \sin \varphi_1 \dots \sin \varphi_{m-2} \cos \varphi_{m-1}$$

$$x_m = \dots \sin \varphi_{m-1}$$

$$J = r^{m-1} (\sin \varphi_1)^{m-2} \dots (\sin \varphi_{m-2})^1$$

Задача 1.

$$\int_0^a dx_1 \int_0^{x_1} dx_2 \int_0^{x_{n-1}} f dx_n = \int_0^a dx_n \int_{x_n}^a dx_{n-1} \int_{x_{n-1}}^a \dots \int_{x_2}^2 dx_1$$

Задача 2.

$$\int_0^t dt_1 \int_0^{t_1} dt_2 \dots \int_0^{t_{n-1}} f(t_1) f(t_2) \dots f(t_n) dt_n = \frac{1}{n!} \left(\int_0^t f(s) ds \right)^n$$

$$\int_0^t f(x_1) \int_0^{x_1} f(x_2) dx_2 dx_1 \stackrel{?}{=} \frac{1}{2} \left(\int_0^t f(x) dx \right)^2 = \frac{1}{2} F(t)^2$$

$$\int_0^t f(x_1) \int_0^{x_1} f(x_2) dx_2 dx_1 = \int_0^t f(x_1) F(x_1) dx_1 = \int_0^t f(x_1) \cdot \frac{1}{(n-1)!} F(t)^{n-1} = \frac{1}{n!} F^n \Big|_0^t$$

Задача 3 (4204a).

$$\int_0^1 \dots \int_0^1 (x_1^2 + \dots + x_n^2) dx_1 \dots dx_n$$

$$\int_0^1 x_k^2 dx_k \int \int \int_{[0,1]} 1 dx_{n-1} = \frac{1}{3} \mapsto \frac{n}{3}$$

Задача 4 (4207).

$$S \sqrt{x_1 + \dots + x_n} dx_1 \dots dx_n$$

$$x_i \geq 0 \quad x_1 + \dots + x_n \leq 1$$

Решение.

$$\begin{cases} t_1 = x_1 \\ t_2 = x_1 + x_2 \\ \vdots \\ t_n = x_1 + x_2 + \dots + x_n \end{cases}$$

$$0 \leq t_1 \leq t_2 \leq \dots \leq t_n \leq 1$$

$$J = 1$$

$$I = \int_0^1 \sqrt{t_n} dt_n \int_0^{t_n} dt_{n-1} \int_0^{t_{n-1}} dt_{n-2} \dots \int_0^{t_2} dt_1$$

$$V_{n-1} = \frac{1}{(n-1)!} R^{n-1}$$

Задача 5 (4210).

$$\frac{x_1^2}{a_1^2} + \dots + \frac{x_{n-1}^2}{a_{n-1}^2} = \frac{x_n^2}{a_n^2} \quad x_n = a_n$$

Решение. Кокус. Замена в полярные

$$x_n = a_n - x$$

$$\begin{cases} x_1 = a_1 r \cos \varphi_1 \\ \vdots \\ x_{n-1} = a_{n-1} r \sin \varphi_1 \dots \sin \varphi_{n-1} \end{cases}$$

$$V = \int_A 1 d\lambda_n = \int_0^1 dx \underbrace{\int_0^x dr \int_0^\pi d\varphi_1 \dots \int_0^{2\pi} d\varphi_{n-2}}_{\lambda_{n-1} B(0,x)} \cdot a_1 a_2 \dots a_n 2^{n-2} (\sin \varphi_2)^{n-3} \dots (\sin \varphi_n - 3)^1 =$$

$$\lambda_m B(0, R) = \frac{\pi^{\frac{m}{2}} R^m}{\Gamma(\frac{m}{2} + 1)}$$

$$= \prod a_i \cdot \int_0^1 dx \cdot \frac{\pi^{\frac{n-1}{2}} R^{n-1}}{\Gamma(\frac{n-1}{2} + 1)} = \prod a_i \cdot \frac{\pi^{\frac{n-1}{2}}}{\Gamma(\frac{n-1}{2} + 1)} \int_0^1 x^{n-1} dx = \frac{\pi^{\frac{n-1}{2}}}{\Gamma(\frac{n-1}{2} + 1) n} \cdot a_1 a_2 \dots a_n$$

Задача 6 (4114).

$$\int_0^x dx_1 \int_0^{x_1} dx_2 \dots \int_0^{x_{n-1}} f(x_n) dx_n = \int_0^x f(u) \frac{(x-u)^{n-1}}{(n-1)!} du$$