

Практика 8

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Содержание

1 Сферические координаты

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$$\begin{aligned}x &= r \cos \phi \cos \psi \\y &= r \sin \phi \cos \psi \quad J = r^2 \\z &= r \sin \psi\end{aligned}$$

Задача 1 (4089).

$$z = x^2 + y^2$$

Решение.

$$r \sin \psi = r^2 \cos^2 \psi \Leftrightarrow r = \frac{\sin \psi}{\cos^2 \psi}$$

$$x = y \Leftrightarrow \phi = \frac{\pi}{4}$$

$$\cos \phi = \sin \phi$$

- $x = 1, r \cos \phi \cos \psi = 1$
- $r = 0, \psi = 0$
- $y = 0, r \sin \phi \cos \psi = 0, \phi = 0$

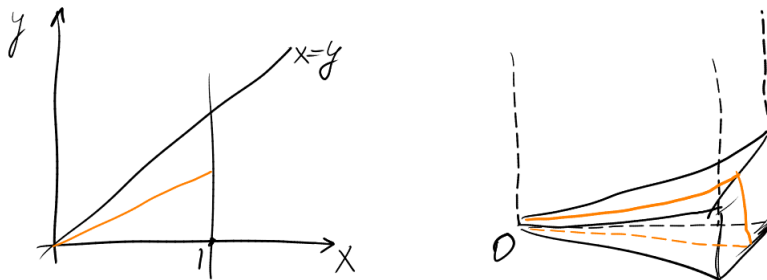


Рис. 1: Фиксируем ϕ

$$\int_0^{\frac{\pi}{4}} d\phi \int_0^{\arctg \frac{1}{\cos \phi}} d\psi \int_0^{\frac{1}{\cos \phi \cos \psi}} f(r) r^2 \cos \psi dr$$

Примечание. **Цилиндрические координаты**

$$\begin{aligned}x &= r \cos \phi \\y &= r \sin \phi \quad J = r \\z &= z\end{aligned}$$

Задача 2 (4118.2).

$$\sqrt[3]{\frac{x}{a}} + \sqrt[3]{\frac{y}{b}} + \sqrt[3]{\frac{z}{c}} = 1 \quad x, y, z \geq 0$$

Решение.

- $x = ar \cos^6 \varphi \cos^6 \psi$
- $y = br \sin^6 \varphi \cos^6 \psi$
- $z = cr \cos^6 \psi$

$$r^{\frac{1}{3}} = 1$$

$$V = \iiint 1 dx dy dz = \int_0^1 dr \int_0^{\frac{\pi}{2}} d\varphi \int_0^{\frac{\pi}{2}} abc \cdot 36 \cdot \cos^5 \psi \sin^5 \psi \cos^{11} \psi \sin^5 \psi$$

Задача 3 (4105).

$$az = a^2 - x^2 - y^2$$

$$z = a - x - y$$

$$x = 0 \quad y = 0 \quad z = 0 \quad a > 0$$

Решение.



$$V = \iiint 1 dx dy dz = \iint dx dy \int_0^{(a^2-x^2-y^2)} 1 dz - \iint dy \int_0^{a-x-y} 1 dz =$$

$$= \int_0^a dr \int_0^{\frac{\pi}{2}} d\varphi \frac{a^2 - r^2}{a} \cdot r - \int_0^a dx \int_0^{a-x} (a - x - y) dy$$

Задача 4 (4108).

$$(x^2 + y^2 + z^2)^2 = a^2(x^2 + y^2 - z^2)$$

Решение.

$$r^4 = a^2(\cos^2 \psi - \sin^2 \psi)r^2$$

$$r = a\sqrt{\cos(2\psi)}$$

$$V = \iiint_{\Omega} 1 dx dy dz = 2 \int_0^{\frac{\pi}{2}} d\psi \int_0^{\sqrt{\cos(2\psi)}} dr \int_0^{2\pi} a^3 \cos \psi \cdot r^2 d\varphi =$$

$$= \frac{4\pi}{3} \int_0^{\frac{\pi}{4}} d\psi \cos \psi \cos^{\frac{3}{2}}(2\psi)$$

Задача 5 (4121).

$$(x^2 + y^2 + z^2)^3 = \frac{a^6 z^2}{x^2 + y^2}$$

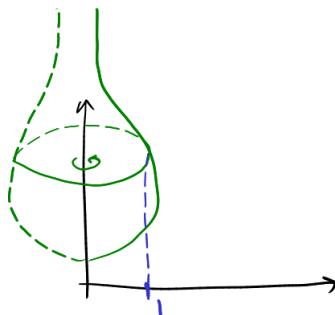
Решение.

- $x = ar \cos \varphi \cos \psi$
- $y = ar \sin \varphi \cos \psi$

• $z = ar \sin \psi$

$$r^6 = \frac{\sin^2 \psi}{\cos^2 \psi}$$

$$r = \sqrt[3]{\operatorname{tg} \psi}$$



$$\begin{aligned} V &= \iiint 1 \, dx \, dy \, dz = 2 \int_0^{\frac{\pi}{2}} d\psi \int_0^{\sqrt[3]{\operatorname{tg} \psi}} dr \int_0^{2\pi} a^3 \cos \psi r^2 d\varphi = \\ &= \int_0^{\frac{\pi}{2}} \frac{2a^3}{3} \cdot 2\pi \operatorname{tg} \psi \cos \psi d\psi = \frac{4a^3\pi}{3} \end{aligned}$$

Задача 6 (4124).

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = \ln \frac{\frac{x}{a} + \frac{y}{b} + \frac{z}{c}}{\frac{x}{a} + \frac{y}{b}}$$

$$x = 0, z = 0, \frac{y}{b} + \frac{z}{c} = 0, \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Решение.

- $u = \frac{x}{a}$
- $v = \frac{x}{a} + \frac{y}{b}$
- $w = \frac{x}{a} + \frac{y}{b} + \frac{z}{c}$

$$J = abc$$

$$w = \ln \frac{w}{v}, u = 0, w - u = 0, w = 1, z = 0 \rightsquigarrow w - v = 0$$

$$w = \ln w - \ln v \Rightarrow v = e^{\ln w - w} = \frac{w}{e^w}$$

$$V = abc \int_0^1 dw \int_0^w du \int_{we^{-w}}^w 1 dv$$

Задача 7 (4012).

Решение. Доделать

Задача 8 (4115).

Решение. Доделать

Задача 9 (4120).

Решение. Доделать

Задача 10 (4123).

Решение. Доделать

Задача 11 (4129).

Решение. Доделать