Практика 8

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Содержание

1 Сферические координаты

$$\begin{aligned} x &= r\cos\phi\cos\psi \\ y &= r\sin\phi\cos\psi \quad J = r^2 \\ z &= r\sin\psi \end{aligned}$$

Задача 1 (4089)**.**

$$z = x^2 + y^2$$

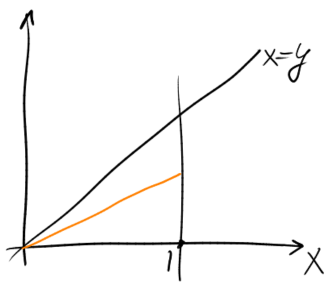
Решение.

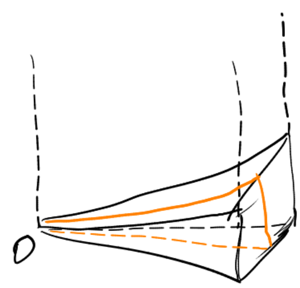
$$r\sin\psi = r^2\cos^2\psi \Leftrightarrow r = \frac{\sin\psi}{\cos^2\psi}$$

$$x = y \Leftrightarrow \varphi = \frac{\pi}{4}$$

$$\cos\varphi = \sin\phi$$

- $x = 1,\ r\cos\varphi\cos\psi = 1$
- $r = 0,\ \psi = 0$
- $y = 0,\ r\sin\varphi\cos\psi = 0,\ \phi = 0$





$$\int\limits_0^{\frac{\pi}{4}} d\varphi \int\limits_0^{\operatorname{arctg}\frac{1}{\cos\phi}} d\psi \int\limits_{\frac{\sin\psi}{\cos^2\psi}}^{\frac{1}{\cos\phi\cos\psi}} f(r) r^2\cos\psi\, dr$$

Замечание. **Цилидрические координаты**

$$\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \quad J = r \\ z &= z \end{aligned}$$

Задача 2 (4118.2)**.**

$$\sqrt[3]{\frac{x}{a}} + \sqrt[3]{\frac{y}{b}} + \sqrt[3]{\frac{z}{c}} = 1 \quad x, y, z \geq 0$$

Решение.

- $x = ar\cos^6\varphi\cos^6\psi$
- $y = br\sin^6\varphi\cos^6\psi$
- $z = cr\cos^6\psi$

$$r^{\frac{1}{3}} = 1$$

$$V = \iiint 1\,dx\,dy\,dz = \int_0^1 dr\int_0^{\frac{\pi}{2}} d\varphi\int_0^{\frac{\pi}{2}} abc\cdot 36\cdot\cos^5\phi\sin^5\phi\cos^{11}\psi\sin^5\psi$$

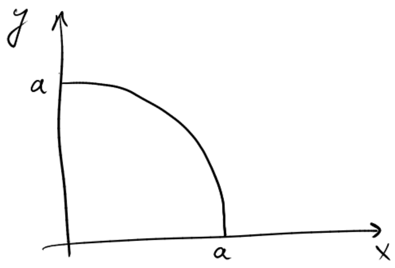
Задача 3 (4105)**.**

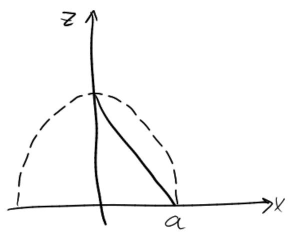
$$az = a^2 - x^2 - y^2$$

$$z = a - x - y$$

$$x = 0 \quad y = 0 \quad z = 0 \quad a > 0$$

Решение.





$$V = \iiint 1\,dx\,dy\,dz = \iint dx\,dy\int\limits_0^{(a^2-x^2-y^2)} 1\,dz - \iint dy\int\limits_0^{a-x-y} 1\,dz =$$

$$= \int_0^a dr\int_0^{\frac{\pi}{2}} d\varphi\frac{a^2-r^2}{a}\cdot r - \int_0^a dx\int\limits_0^{a-x}(a-x-y)\,dy$$

Задача 4 (4108)**.**

$$(x^2+y^2+z^2)^2 = a^2(x^2+y^2-z^2)$$

Решение.

$$r^4 = a^2(\cos^2\psi - \sin^2\psi)r^2$$

$$r = a\sqrt{\cos(2\psi)}$$

$$V = \iiint_\Omega 1\,dx\,dy\,dz = 2\int_0^{\frac{\pi}{2}} d\psi\int\limits_0^{\sqrt{\cos(2\psi)}} dr\int_0^{2\pi} a^3\cos\psi\cdot r^2 d\varphi =$$

$$= \frac{4\pi}{3}\int_0^{\frac{\pi}{4}} d\psi\cos\psi\cos^{\frac{3}{2}}(2\psi)$$

Задача 5 (4121)**.**

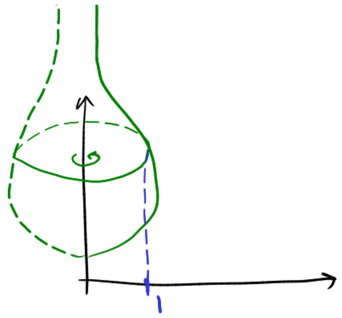
$$(x^2+y^2+z^2)^3 = \frac{a^6z^2}{x^2+y^2}$$

Решение.

- $x = ar\cos\varphi\cos\psi$
- $y = ar\sin\varphi\cos\psi$
- $z = ar\sin\psi$

$$r^6 = \frac{\sin^2\psi}{\cos^2\psi}$$

$$r = \sqrt[3]{\operatorname{tg}\psi}$$



$$V = \iiint 1\,dx\,dy\,dz = 2\int\limits_0^{\frac{\pi}{2}} d\psi \int\limits_0^{\sqrt[3]{\operatorname{tg}\psi}} dr \int\limits_0^{2\pi} a^3\cos\psi r^2 d\varphi =$$

$$= \int_0^{\frac{\pi}{2}} \frac{2a^3}{3}\cdot 2\pi \operatorname{tg}\psi\cos\psi\,d\psi = \frac{4a^3\pi}{3}$$

Задача 6 (4124)**.**

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = \ln\frac{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}}{\frac{x}{a}+\frac{y}{b}}$$

$$x=0,\ z=0,\ \frac{y}{b}+\frac{z}{c}=0,\ \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Решение.

- $u = \frac{x}{a}$
- $v = \frac{x}{a}+\frac{y}{b}$
- $w = \frac{x}{a}+\frac{y}{b}+\frac{z}{c}$

$$J = abc$$

$$w = \ln\frac{w}{v},\ u=0,\ w-u=0,\ w=1,\ z=0 \rightsquigarrow w-v=0$$

$$w = \ln w - \ln v \Rightarrow v = e^{\ln w - w} = \frac{w}{e^w}$$

$$V = abc\int_0^1 dw \int_0^w du \int\limits_{we^{-w}}^{w} 1dv$$

Задача 7 (4012)**.**

Решение. Доделать

Задача 8 (4115)**.**

Решение. Доделать

Задача 9 (4120)**.**

Решение. Доделать

Задача 10 (4123)**.**

Решение. Доделать

Задача 11 (4129)**.**

Решение. Доделать