

Практика 3

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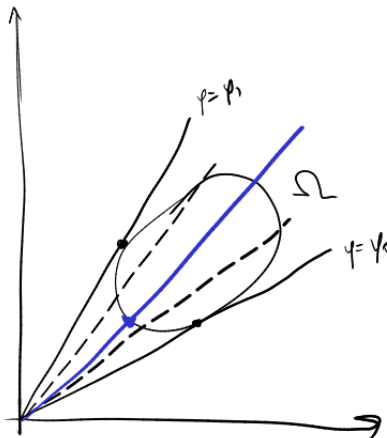
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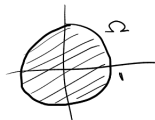
1 Полярные координаты

- $x = r \cos \varphi$
- $y = r \sin \varphi$

$$\iint_{\Omega} f dx dy = \int_{\varphi_0}^{\varphi_1} d\varphi \int_{r_0(\varphi)}^{r_1(\varphi)} f(r \cos \varphi, r \sin \varphi) \cdot J dr = \int_{r_0}^{r_1} dr \int_{\varphi_0(r)}^{\varphi_1(r)} f \cdot J d\varphi$$

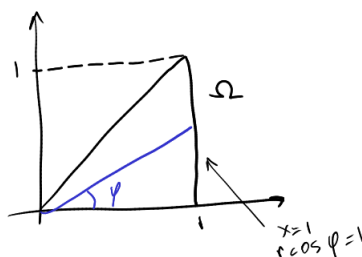


Задача 1.



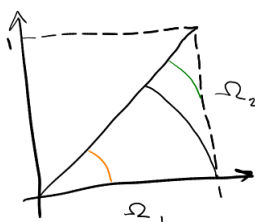
$$\iint_{\Omega} f dx dy = \int_0^{2\pi} d\varphi \int_0^1 f \cdot J dr$$

Задача 2.



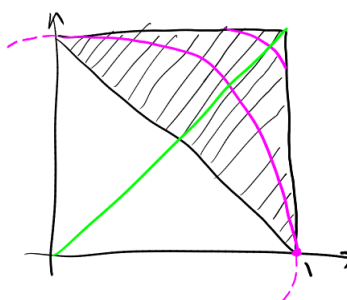
$$\iint_{\Omega} f dx dy = \int_0^{\frac{\pi}{4}} d\varphi \int_0^{\frac{1}{\cos \varphi}} dr f \cdot J = \int dr \int d\varphi f \cdot J$$

Задача 3.



$$\iint_{\Omega} f dx dy = \int_0^1 dr \int_0^{\frac{\pi}{4}} d\varphi f \cdot J + \int_1^{\sqrt{2}} dr \int_{\arccos \frac{1}{2}}^{\frac{\pi}{4}} d\varphi f \cdot J$$

Задача 4.



$$\begin{aligned} \iint_{\Omega} &= \int_0^{\frac{\pi}{4}} d\varphi \int_{\frac{1}{\cos \varphi + \sin \varphi}}^{\frac{1}{\cos \varphi}} dr \dots + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int_{\frac{1}{\cos \varphi + \sin \varphi}}^{\frac{1}{\sin \varphi}} dr \dots = \\ &= \int_{\frac{\sqrt{2}}{2}}^1 dr \int \dots \end{aligned}$$

Задача 5.

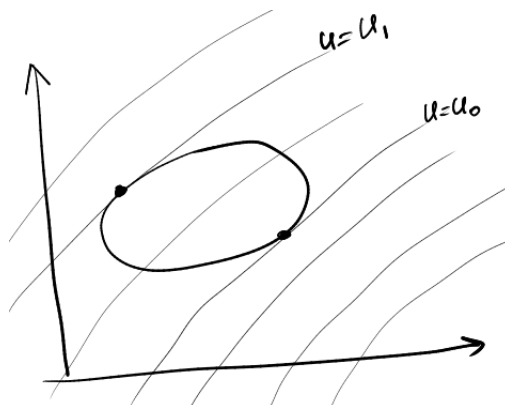
$$x^2 + y^2 \leq \alpha x$$

$$r = \alpha \cos \varphi$$

$$\iint_{\Omega} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int_0^{\alpha \cos \varphi} f \cdot J dr = \int_0^{\alpha} dr \int_{-\arccos \frac{r}{\alpha}}^{\arccos \frac{r}{\alpha}} f \cdot J d\varphi$$

2 Замена переменных

- $x = x(u, v)$
- $y = y(u, v)$



Фиксируем $u = u_0$:

$$\begin{cases} x = x(u_0, v) \\ y = y(u_0, v) \end{cases}$$
$$\iint_{\Omega} f dx dy = \int_{u_0}^{u_1} du \int_{v_0(u)}^{v_1(u)} f(x(u, v), y(u, v)) \left(\det \begin{pmatrix} x'_u & x'_v \\ y'_u & y'_v \end{pmatrix} \right) dv$$
$$\int f(x) dx = \int f(x(t)) x' dt$$