

# Практика 10

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## Содержание

### Задача 1.

$$\int_{\Omega_+} \langle F, n_0 \rangle ds$$

Решение.

- $F = (P, Q, R)$
- $\Phi : G \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3$

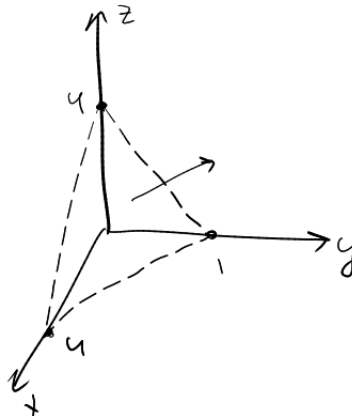
$$\begin{aligned} I &= \int \left\langle F, \frac{\Phi'_u \times \Phi'_v}{|\Phi'_u \times \Phi'_v|} \right\rangle \cdot |\Phi'_u \times \Phi'_v| du dv = \int_G \langle F, \Phi'_u \times \Phi'_v \rangle = \\ &= \int_G P \begin{vmatrix} y'_u & y'_v \\ z'_u & z'_v \end{vmatrix} + Q \begin{vmatrix} z'_u & z'_v \\ x'_u & x'_v \end{vmatrix} + R \begin{vmatrix} x'_u & x'_v \\ y'_u & y'_v \end{vmatrix} du dv = \\ &= \int_G P dy dz + Q dz dx + R dx dy \\ &\quad \Phi = (x(u, v), y(u, v), z(u, v)) \\ \langle F, \Phi'_u \times \Phi'_v \rangle &= \det \begin{pmatrix} P & x'_u & x'_v \\ Q & y'_u & y'_v \\ R & z'_u & z'_v \end{pmatrix} \end{aligned}$$

### Задача 2.

$$\iint (2z - x) dy dx + (x + 2z) dz dx + 3z dx dy$$

- $x + 4y + z = 4$
- $z, y, z \geq 0$
- верхняя сторона

Решение.

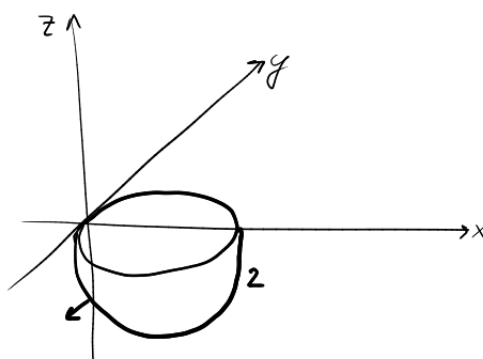


$$\begin{aligned} \iint \dots &= \iint \left\langle \begin{pmatrix} 2z - x \\ x + 2z \\ 3z \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{18}} \\ \frac{4}{\sqrt{18}} \\ \frac{1}{\sqrt{18}} \end{pmatrix} \right\rangle ds = \frac{1}{\sqrt{18}} \iint (13z + 3x) ds \\ &= \frac{1}{\sqrt{18}} \int_0^4 dx \int_0^{1-\frac{x}{4}} dy 13(4 - x - 4y) + 3x\sqrt{18} \end{aligned}$$

**Задача 3.**

$$\iint_{S_+} (x - 1)^2 dy dz + 0 dz dx + 0 dx dy$$

- $x^2 + y^2 + z^2 = 2x$
- $z \leq 0$
- верхняя сторона



Решение.

- $x = x$
- $y = y$
- $z = -\sqrt{2x - x^2 - y^2}$

$$\begin{aligned} x'_x &= 1 & x'_y &= 0 \\ y'_x &= 0 & y'_y &= 1 \\ z'_x &= \frac{x-1}{\sqrt{-x^2-y^2+2x}} & z'_y &= \frac{y}{\sqrt{-x^2-y^2+2x}} \end{aligned}$$

$$\iint_0^2 dx \int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} (x-1)^2 \left| \begin{matrix} 0 & 1 \\ \frac{x-1}{\sqrt{-x^2-y^2+2x}} & \frac{y}{\sqrt{-x^2-y^2+2x}} \end{matrix} \right| = \iint (x-1)^2 \left( -\frac{(x-1)}{\sqrt{-x^2-y^2+2x}} \right)$$

Решение. Немного не та задача

- $x = 1 + r \cos \varphi \cos \psi$
- $y = 2 \sin \varphi \cos \psi$
- $z = r \sin \psi$

$$\frac{\partial}{\partial \varphi}$$

$$\begin{aligned} -r \sin \varphi \cos \psi \\ r \cos \varphi \cos \psi \\ 0 \end{aligned}$$

$$\frac{\partial}{\partial \psi}$$

$$\begin{aligned} -r \cos \varphi \sin \psi \\ -r \sin \varphi \sin \psi \\ r \cos \psi \end{aligned}$$

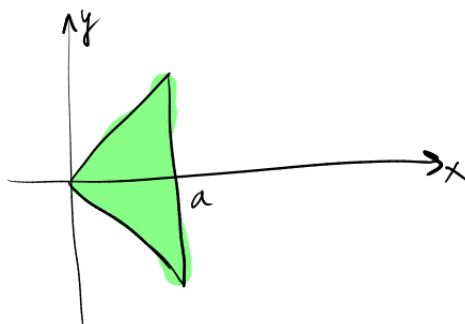
$$\Phi'_u \times \Phi'_v : \begin{matrix} r \cos \varphi \cos^2 \psi \\ r \sin \varphi \cos^2 \psi \\ r \sin \psi \cos \psi \end{matrix}$$

$$\begin{aligned} \iint \dots &= \int_0^{2\pi} d\varphi \int_{-\frac{\pi}{2}}^0 d\psi \cos \varphi \cos \psi \cdot \cos \varphi \cos^2 \psi = \\ &= \pi \int_{-\frac{\pi}{2}}^0 (1 - \sin^2 \psi) d \sin \psi = \pi \left( s - \frac{s^3}{3} \right) \Big|_{-1}^0 \end{aligned}$$

**Задача 4.**

$$\iint x \, dy \, dz + y \, dz \, dx + z \, dx \, dy$$

- $z = x^2 - y^2$
- $|y| \leq x \leq a$
- верхняя сторона



*Решение.*

$$\Phi = \begin{pmatrix} x \\ y \\ x^2 - y^2 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 2x & -2y \end{pmatrix}$$

$$\iint x(-2x) + y(2y) + (x^2 - y^2) \cdot 1 \, dx \, dy$$