Практика 10

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Содержание

Задача 1.

$$\int_{\Omega_+} \langle F, n_0 \rangle \, ds$$

Решение.

- $F = (P, Q, R)$
- $\Phi : G \subset \mathbb{R}^2 \to \mathbb{R}^3$

$$I = \int \left\langle F, \frac{\Phi'_u \times \Phi'_v}{|\Phi'_u \times \Phi'_v|} \right\rangle \cdot |\Phi'_u \times \Phi'_v| \, du \, dv = \int_G \langle F, \Phi'_u \times \Phi'_v \rangle =$$

$$= \int_G P \begin{vmatrix} y'_u & y_v \\ z'_u & z'_v \end{vmatrix} + Q \begin{vmatrix} z'_u & z'_v \\ x'_u & x'_v \end{vmatrix} + R \begin{vmatrix} x'_u & x'_v \\ y'_u & y'_v \end{vmatrix} \, du \, dv =$$

$$= \int_G P \, dy \, dz + Q \, dz \, dx + R \, dx \, dy$$

$$\Phi = (x(u, v), y(u, v), z(u, v))$$

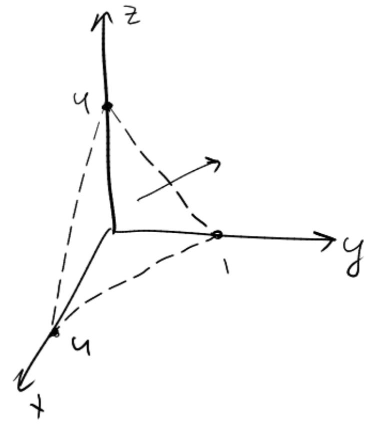
$$\langle F, \Phi'_u \times \Phi'_v \rangle = \det \begin{pmatrix} P & x'_u & x'_v \\ Q & y'_u & y'_v \\ R & z'_u & z'_v \end{pmatrix}$$

Задача 2.

$$\iint (2z - x) \, dy \, dx + (x + 2z) \, dz \, dx + 3z \, dx \, dy$$

- $x + 4y + z = 4$
- $z, y, z \geq 0$
- верхняя сторона

Решение.



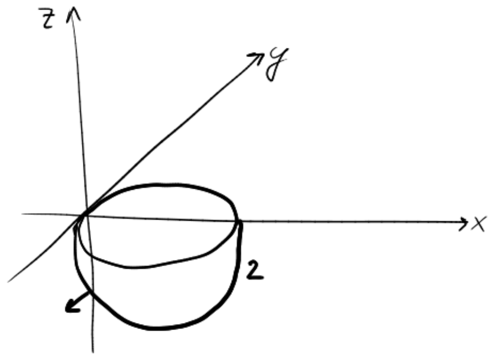
$$\iint \cdots = \iint \left\langle \begin{pmatrix} 2z - x \\ x + 2z \\ 3z \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{18}} \\ \frac{4}{\sqrt{18}} \\ \frac{1}{\sqrt{18}} \end{pmatrix} \right\rangle ds = \frac{1}{\sqrt{18}} \iint (13z + 3x)\, ds$$

$$= \frac{1}{\sqrt{18}} \int_0^4 dx \int_0^{1-\frac{x}{4}} dy 13(4 - x - 4y) + 3x\sqrt{18}$$

Задача 3.

$$\iint_{S_+} (x-1)^2\, dy\, dz + 0\, dz\, dx + 0\, dx\, dy$$

- $x^2 + y^2 + z^2 = 2x$
- $z \leq 0$
- верхняя сторона



Решение.

- $x = x$
- $y = y$
- $z = -\sqrt{2x - x^2 - y^2}$

$$\begin{matrix} x'_x = 1 & x'_y = 0 \\ y'_x = 0 & y'_y = 1 \\ z'_x = \frac{x-1}{\sqrt{-x^2-y^2+2x}} & z'_y = \frac{y}{\sqrt{-x^2-y^2+2x}} \end{matrix}$$

$$\iint_0^2 dx \int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} (x-1)^2 \begin{vmatrix} 0 & 1 \\ \frac{x-1}{\sqrt{-x^2-y^2+2x}} & \frac{y}{\sqrt{-x^2-y^2+2x}} \end{vmatrix} = \iint (x-1)^2 \left(-\frac{(x-1)}{\sqrt{-x^2-y^2+2x}} \right)$$

Решение. Немного не та задача

- $x = 1 + r\cos\varphi\cos\psi$
- $y = 2\sin\varphi\cos\psi$
- $z = r\sin\psi$

$\frac{\partial}{\partial\varphi}$

$$\begin{matrix} -r\sin\varphi\cos\psi \\ r\cos\varphi\cos\psi \\ 0 \end{matrix}$$

$\frac{\partial}{\partial\psi}$

$$\begin{matrix} -r\cos\varphi\sin\psi \\ -r\sin\varphi\sin\psi \\ r\cos\psi \end{matrix}$$

$$\Phi'_u \times \Phi'_v : \begin{matrix} r\cos\varphi\cos^2\psi \\ r\sin\varphi\cos^2\psi \\ r\sin\psi\cos\psi \end{matrix}$$

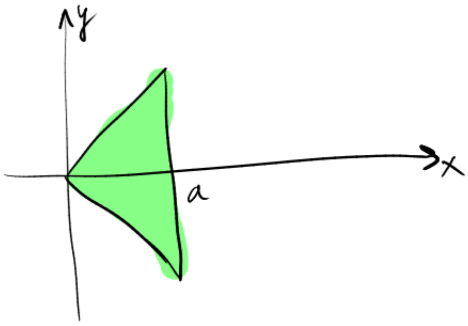
$$\iint \cdots = \int_0^{2\pi} d\varphi \int_{-\frac{\pi}{2}}^{0} d\psi \cos\varphi\cos\psi \cdot \cos\varphi\cos^2\psi =$$

$$= \pi \int_{-\frac{\pi}{2}}^{0} (1-\sin^2\psi) d\sin\psi = \pi \left(s - \frac{s}{3}\right)^3 \Big|_{-1}^{0}$$

Задача 4.

$$\iint x\,dy\,dz + y\,dz\,dx + z\,dx\,dy$$

- $z = x^2 - y^2$
- $|y| \le x \le a$
- верхняя сторона



Решение.

$$\Phi = \begin{pmatrix} x \\ y \\ x^2 - y^2 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 2x & -2y \end{pmatrix}$$

$$\iint x(-2x) + y(2y) + (x^2 - y^2) \cdot 1\,dx\,dy$$