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Содержание

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$$\begin{aligned}
 f_{n+2} &= f_{n+1} + f_n \\
 f_{n+1} &= f_n + f_{n-1} \Rightarrow f_{n-1} = f_{n+1} - f_n \\
 f_{n+2}^2 + f_{n-1}^2 &= f_{n+1}^2 + 2f_{n+1}f_n + f_n^2 + f_{n+1}^2 - 2f_{n+1}f_n + f_n^2 = 2f_{n+1}^2 + 2f_n^2 \\
 f_{n+2}^2 &= 2f_{n+1}^2 + 2f_n^2 - f_{n-1}^2 \\
 f_n^2 &= 2f_{n-1}^2 + 2f_{n-2}^2 - f_{n-3}^2 \\
 g_n &:= f_n^2 \\
 g_n &= 2g_{n-1} + 2g_{n-2} - g_{n-3}
 \end{aligned}$$

Находим производящую функцию:

- $c_1 = 2$
- $c_2 = 2$
- $c_3 = -1$
- $g_0 = f_0^2 = 1$
- $g_1 = f_1^2 = 1$
- $g_2 = f_2^2 = 4$
- $p_0 = g_0 = 1$
- $p_1 = g_1 - g_0c_1 = 1 - 2 \cdot 1 = -1$
- $p_2 = g_2 - (g_1c_1 + g_0c_2) = 4 - (2 + 2) = 0$

Итого

$$A(t) = \frac{1-t}{1-2t-2t^2+t^3}$$